ULTRASOUND SIMULATION FOR 3D-AXISYMMETRIC MODELS

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Abstract - Development of ultrasound nondestructive evaluation testing (NDT) techniques has involved a combination of both analytic and experimental methods. In contrast, relatively little software exists for ultrasound simulation in NDT, particularly in comparison to that existing for stress-strain and electromagnetic areas. This paper describes new software for simulating the full (longitudinal and shear) solution to the three-dimensional (3D) axisymmetric wave equation. The simulation software is able to model both liquids and solids, and also accounts for losses using a classic viscoelastic model. The program computes the solution using a finite difference time domain algorithm, and evaluates the displacement vector at each (discrete grid) point of the object. Sources and receivers may be placed anywhere in or on the object, which is assumed to be cylindrical. A comparison of the on-axis diffraction pattern results obtained with the simulation software show excellent agreement with analytic results. Results using a cladded rod are also presented. This software should help to broaden the use of computational methods in ultrasonic NDT and in ultrasonics in general.

I. INTRODUCTION

The use of computer simulation is a common tool in a variety of engineering disciplines and problems. In terms of commercial software, most such applications involve either electromagnetics or structural analysis. With respect to ultrasonics, there is little commercial software available for the engineer to use, although there is no doubt a need for such computational methods in this field as well. Although many specialized software implementations have been described, much of this is either proprietary, or an academic package that is very difficult to use and often with little or no support or documentation.

For the past several years a software package known as *Wave2000* has been commercially available and is widely used across a spectrum of applications. However, it is limited to two-dimensional (2D) objects only (that is objects with constant cross-sections). In an effort to address this limitation, a new software package known as *Wave2500* has been developed. *Wave2500* computes the solution to the 3D-axisymmetric viscoelastic wave equation. The purpose of this paper is to introduce this new software tool to the ultrasound engineering community, and to show through examples some of *Wave2500's* capabilities and features.

II. PROGRAM DESCRIPTION

Wave2500 is a stand alone software package for computational ultrasonics. It operates by solving the (3D) viscoelastic wave equation in the time domain. The solution is computationally intensive; however, thanks to the ever increasing power and speed of computer hardware, *Wave2500* is a practical tool for practical problems. *Wave2500* can provide solutions to a reasonably broad range of 3D ultrasound problems. The program allows the user to specify an object to be ultrasonically interrogated. By definition, the object itself must be axisymmetric; that is, no change in material properties are allowed as a function of the cylindrical coordinate ϕ, when r and z are held constant. The object is specified as a pixel based graphics file., composed of individual pixels which can have 1 of 256 (0-255) gray levels. Each pixel value represents a physical material (e.g., water, steel, etc.) that is set by the user. Gray level 255 is reserved to denote void. There is thus a vast variety of 3D structures which can be simulated using *Wave2500*. The 3D object can be generated either internally using *Wave2500's* "Geometry" routines or externally using virtually any graphics program which can output files in 8 bit monochrome format. The image also may be obtained from various scan modalities, for example CT, MRI or microscopic histological “slice” data, which has been converted to a suitable graphics file format [1]. Usually, a segmentation algorithm of some kind would be necessary to properly associate various regions of the image with a particular material (i.e., grey level).

The attractiveness of *Wave2500* is that it makes it very easy to generate solutions to a wide variety of 3D ultrasound problems within a simple graphical interface. The user has access to many features designed to mimic reasonably closely many practical situations. For example, there are source and receiver configurations that are similar to real ultrasound experimental configurations, for example the use of signals that characterize many transducer generated waveforms. *Wave2500* has the potential to generate new insights and approaches to many problems in ultrasonics. The user has the capability to "experiment" at the computer, without turning on a pulser-receiver or connecting a "BNC" cable. The computer can work "around the clock" computing solutions to problems that are difficult to perform in the laboratory or field, but the results of the simulations can provide important understanding and knowledge for future experiments or for data already collected.

Wave2500 computes an approximate solution to the 3D-axisymmetric viscoelastic wave equation. The numerical solution is obtained via a finite difference method [2]. The 3D-axisymmetric solution is based on the 3D elastic vector wave equation expressed in cylindrical coordinates, in which there is assumed to be *no dependence on the angular coordinate, ϕ*. Then, the axisymmetric (lossless) wave equation can be written as the following set of 2 scalar equations:

$$\rho\frac{\partial^2 u_r}{\partial t^2}=(\lambda+2\mu)\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru_r)\right)+\mu\frac{\partial^2 u_r}{\partial z^2}+(\lambda+\mu)\frac{\partial^2 u_z}{\partial z\partial r}$$

$$\rho\frac{\partial^2 u_z}{\partial t^2}=(\lambda+\mu)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial z}\right)+(\lambda+2\mu)\frac{\partial^2 u_z}{\partial z^2}+\mu\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) \qquad (1)$$

In Eq. (1) above, ρ is the mass per unit volume or material density in units of kg/m^3, λ and μ are the first and second Lamé constants, respectively, both in units of N/m^2, ∂ denotes the partial differential operator, u_r and u_z are the spatial and time-dependent displacements in the *r* and *z* cylindrical coordinate directions, respectively, both in units of meter, and t is time in seconds. Note that the mass and Lamé constants are functions of location, in the case of a heterogeneous object, but this dependence is not explicitly shown above in Eq. (1). The axisymmetric case assumes that the object and sources are symmetric with respect to the angular coordinate, ϕ; this leads to the angular displacement, u_ϕ, being identically zero. Also note that in this axisymmetric problem the displacement $u_r = 0$ for r = 0, for any value of z.

Equation (1) needs to be generalized to include the effects of dissipation; a viscous loss mechanism as described in [3] is utilized. Briefly, the viscous loss term is obtained by introduction of the viscous loss tensor, η_{klmn}:

$$\eta_{klmn}=\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22}\end{bmatrix} \qquad \textbf{(2a)}$$

where each submatrix N_{ij} is a 3x3 matrix, $N_{12} = N_{21} = 0$, and

$$N_{11}=\begin{bmatrix} \xi+4/3\eta & \xi-2/3\eta & \xi-2/3\eta \\ \xi-4/3\eta & \xi+2/3\eta & \xi-2/3\eta \\ \xi-4/3\eta & \xi-2/3\eta & \xi+2/3\eta \end{bmatrix} \qquad \textbf{(2b)}$$

$$N_{22}=\begin{bmatrix} \eta & 0 & 0 \\ 0 & \eta & 0 \\ 0 & 0 & \eta \end{bmatrix} \qquad \textbf{(2c)}$$

where η is the shear (sometimes also called the 1st) viscosity, and ξ (denoted by “ϕ” in the *Wave2500* “Material Property” page) is the bulk (sometimes also called the 2nd) viscosity, both in units of $N\text{-}s/m^2$. The viscosity tensor of Eq. (2) is added to the standard elastic stiffness tensor to obtain the viscoelastic (lossy) version of the lossless elastic wave equation (Eq. (1)). Using this viscous loss formulation, a new set of Lamé "constants" can be derived:

$$\lambda \rightarrow \lambda + (\xi - \frac{2}{3}\eta)\frac{\partial}{\partial t} \quad \textbf{(3a)}$$

$$\mu \rightarrow \mu + \eta \frac{\partial}{\partial t} \quad \textbf{(3b)}$$

where the viscous loss component is seen to be contained in the partial derivative operator. This can be related in the frequency domain to real and imaginary parts for the two Lamé constants, respectively, by associating the partial derivative term with jω, where j = √-1 and ω is radian frequency, and using the frequency domain to derive the longitudinal and shear mode propagation constants (attenuation and phase velocity) associated with the wave equation. For example, the shear attenuation and velocity are given by:

$$\alpha_T = \frac{\omega}{\sqrt{2}} \sqrt{\frac{\rho}{(\mu_r^2 + \mu_i^2)}} \frac{\mu_i}{\sqrt{\mu_r + \sqrt{\mu_r^2 + \mu_i^2}}} \quad (4a)$$

$$v_T = \sqrt{\frac{(\mu_r^2 + \mu_i^2)}{\rho}} \frac{\sqrt{2}}{\sqrt{\mu_r + \sqrt{\mu_r^2 + \mu_i^2}}} \quad (4b)$$

In Eq. (4), the subscripts *"r"* and *"i"* denote real and imaginary parts of the Lamé constants in the frequency domain, respectively, v_T and α_T denote transverse velocity and attenuation, respectively, all as a function of radian frequency, $\omega = 2\pi f$. Analogous equations exist for the longitudinal propagation constants, and may be found in [4]. These variables are displayed on the Material Property page at a prescribed value for frequency, *f*.

As noted above, standard finite difference methods are used to obtain the numerical solutions. In particular, *Wave2500* uses a space-centered second order difference approximation for the *r* and *z* derivatives, and a second order forward difference for the time derivative. It is of particular interest to note that the computational complexity of the 3D-axisymmetric problem is similar to that of the 2D case. This is extremely beneficial in that ultrasound propagation through true 3D (albeit axisymmetric) objects can be simulated with about the same overhead as the 2D case, which has already been shown to result in solutions in a matter of minutes, for "practical sized objects" [1].

III. Examples

Two examples using *Wave2500* are provided here to illustrate some of the main features of the program. The first simulation demonstrates the close equivalence of a *Wave2500* simulation to a classic analytic result for circular transducers, the on-axis displacement amplitude.

Validation

In this simulation, the on-axis displacement amplitude in water due to a 1 MHz continuous sine wave with a 1 cm radius circular transducer was simulated. The transducer was centered on a cylinder of 4 cm radius and 20 cm in length. Boundary conditions were set to simulate an infinite cylinder of water (approximately zero reflections at the bottom and outside of the cylinder). The theoretical curve in Fig. 1 was computed using Equation 3.2.4 of [5]. As may be seen from Fig. 1, there is excellent correspondence between the simulated data and the analytic data.

Cladded Rod Example

In this example, a steel rod 10 mm in radius was cladded with a Plexiglas tube of 20 mm outer diameter. The overall length of the cladded cylinder was 30 mm. A circular transducer of 10 mm radius was centered on the cladded rod, namely directly over and covering completely the steel rod but not over any part of the plastic. The source transducer emitted a sine wave with a Gaussian envelope, centered at 1 MHz. Shown in Fig. 2 is a "snapshot" of the propagating wave at t= 4.8 μs, with the grey level proportional to the amplitude of the displacement vector magnitude. As may be seen, there are a number of distinct modes propagating. It is also possible to define a number of receiver transducers, and plots of the received signals can be defined. The receiver data can also be saved to files for later analysis. It is also possible to assign time delays to an annular source or receiver array for focusing at a given depth.

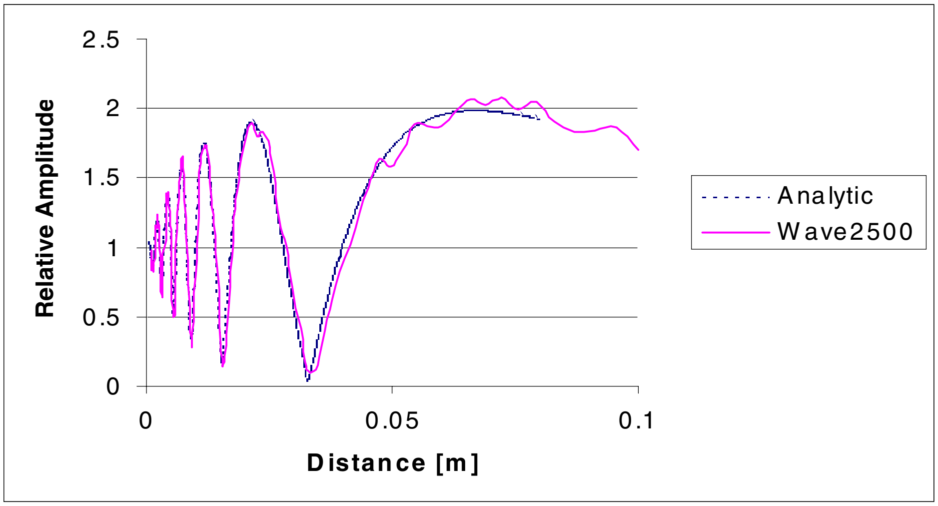

Figure 1. Theoretical and simulated on-axis displacement amplitude of a 1 cm circular source at 1 MHz.

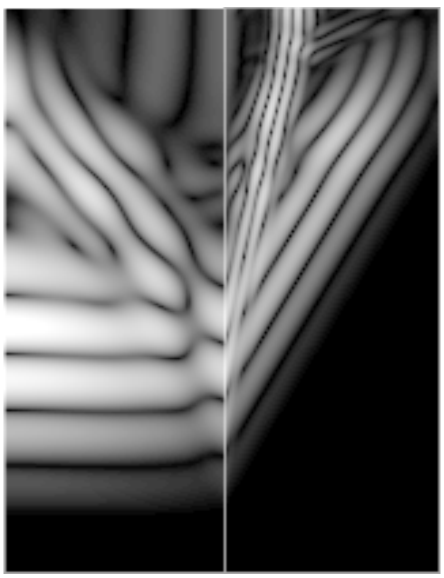

Figure 2. Snapshot of a propagating wave in a steel rod cladded with plastic, at time t = 4.8 microsecond. See text for additional details.

IV. CONCLUSION

This paper has provided a brief introduction into the use of *Wave2500* for simulation of 3D-axisymmetric models. While simulation cannot replace analytical or experimental studies, it can provide important insights that can lead to many practical advantages. The description of the *Wave2500* software presented should lead to the use of simulation in cases when a 3D-axisymmetric model is appropriate.

V. REFERENCES

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